DINAKRUSHNA COLLEGE, JALESWAR
DEPARTMENT OF PHYSICS
NEP–2020 UNDERGRADUATE PROGRAMME
B.Sc. PHYSICS – SEMESTER I
PAPER–I: MATHEMATICAL PHYSICS–I
UNIT-WISE NUMERICAL QUESTION BANK

UNIT-I: CALCULUS

1-Mark Questions

1. Evaluate
$$\lim_{x\to 0}\frac{\sin x}{x}.$$
2. Find
$$\frac{d}{dx}(x^3e^x).$$
3. Find
$$\frac{d}{dx}(\ln x).$$
4. Evaluate
$$\int x^2\,dx.$$
5. Find the integrating factor of
$$\frac{dy}{dx}+3y=x.$$
6. Find the Wronskian
$$W(e^x,e^{2x}).$$
7. Find $\partial f/\partial x$ for
$$f(x,y)=x^2y+y^3.$$
8. Find the stationary point of
$$f(x,y)=x^2+y^2.$$
9. Write the auxiliary equation of
$$y''-5y'+6y=0.$$
10. Find the first two terms of the binomial expansion of
$$(1+x)^{1/2}.$$

2-Mark Questions

1. Solve
$$\frac{dy}{dx}+2y=0.$$
2. Solve
$$\frac{dy}{dx}=3x^2-2x.$$

3. Find the integrating factor of
$$\frac{dy}{dx} + \frac{2}{x}y = x^2.$$
4. Find the Wronskian
$$W(x, x^2).$$
5. Find $\partial f/\partial x$ and $\partial f/\partial y$ for
$$f(x, y) = x^2y + xy^2.$$
6. Test whether
$$(2xy + y^2)\,dx + (x^2 + 2xy)\,dy = 0$$
is exact.
7. Expand $(1 + x)^3$ using the binomial theorem.
8. Find the stationary points of
$$f(x, y) = x^2 - 4x + y^2 - 6y.$$
9. Solve
$$y'' - 3y' + 2y = 0.$$
10. Find the Taylor expansion of e^x up to the x^2 term.

5-Mark Questions

1. Solve
$$\frac{dy}{dx} + 2y = e^x.$$
2. Solve
$$\frac{dy}{dx} - \frac{2}{x}y = x^2.$$
3. Solve
$$y'' - 5y' + 6y = 0.$$
4. Solve
$$y'' + 4y = 0,$$
given $y(0) = 1$ and $y'(0) = 0$.
5. Solve
$$y'' - 4y = xe^x.$$
6. Find the Taylor series of $\ln(1 + x)$ up to x^5.
7. Find the maximum and minimum of
$$f(x, y) = x^2 + y^2 - 4x - 6y.$$
8. Using Lagrange multipliers, find the extrema of xy subject to
$$x + y = 10.$$
9. Find the Wronskian and test the linear independence of x and xe^x.

8-Mark Questions

1. Solve
$$y'' - 7y' + 12y = e^{2x},$$
showing the complementary function and particular integral.
2. Solve
$$y'' + 4y' + 4y = x^2 + 1.$$
3. Solve
$$y'' - y = e^{2x} + x.$$
4. Using Lagrange multipliers, find the maximum value of xyz subject to
$$x + y + z = 12, \qquad x, y, z > 0.$$
5. Find the extrema of
$$f(x, y) = x^2y + y^2x - 4xy$$
and classify the stationary points.
6. Obtain the Taylor expansion of $\ln(1 + x)$ and use it to estimate
$$\ln(1.1)$$
correct to four decimal places.

UNIT-II: VECTOR ALGEBRA AND VECTOR DIFFERENTIATION

1-Mark Questions

1. Find $\mathbf{A} \cdot \mathbf{B}$ for
$$\mathbf{A} = \hat{\imath} + \hat{\jmath}, \qquad \mathbf{B} = \hat{\imath} - \hat{\jmath}.$$
2. Find $|\mathbf{A}|$ for
$$\mathbf{A} = 3\hat{\imath} + 4\hat{\jmath}.$$
3. Find $\mathbf{A} \times \mathbf{B}$ for
$$\mathbf{A} = \hat{\imath}, \qquad \mathbf{B} = \hat{\jmath}.$$
4. Find
$$|\hat{\imath} + \hat{\jmath} + \hat{k}|.$$
5. Find $\nabla \cdot \mathbf{A}$ for
$$\mathbf{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}.$$
6. Find $\nabla \times \mathbf{A}$ for
$$\mathbf{A} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}.$$
7. Find
$$\nabla(x^2 + y^2 + z^2).$$
8. State the condition for a vector field to be solenoidal.
9. State the condition for a vector field to be irrotational.
10. Find the angle between two perpendicular unit vectors.

2-Mark Questions

1. Find $\mathbf{A} \cdot \mathbf{B}$ for
$$\mathbf{A} = 2\hat{i} - \hat{j} + 3\hat{k}, \qquad \mathbf{B} = \hat{i} + 2\hat{j} - \hat{k}.$$
2. Find $\mathbf{A} \times \mathbf{B}$ for
$$\mathbf{A} = \hat{i} + 2\hat{j} + \hat{k}, \qquad \mathbf{B} = 2\hat{i} - \hat{j} + 3\hat{k}.$$
3. Find the angle between
$$\mathbf{A} = \hat{i} + \hat{j} + \hat{k}$$
and
$$\mathbf{B} = \hat{i} - \hat{j}.$$
4. Find the area of the parallelogram formed by
$$\mathbf{A} = 2\hat{i} + \hat{j}, \qquad \mathbf{B} = \hat{i} + 3\hat{j}.$$
5. Evaluate the scalar triple product
$$[\mathbf{A}\,\mathbf{B}\,\mathbf{C}]$$
for
$$\mathbf{A} = \hat{i} + \hat{j} + \hat{k},$$
$$\mathbf{B} = 2\hat{i} - \hat{j} + \hat{k},$$
$$\mathbf{C} = \hat{i} + 2\hat{j} - \hat{k}.$$
6. Find $\nabla\phi$ for
$$\phi = x^2 y + yz^2.$$
7. Find $\nabla \cdot \mathbf{A}$ for
$$\mathbf{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}.$$
8. Find $\nabla \times \mathbf{A}$ for
$$\mathbf{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}.$$
9. Determine whether
$$\mathbf{A} = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$
is solenoidal.
10. Find the directional derivative of
$$\phi = x^2 + y^2$$
at $(1, 2)$ in the direction $\hat{i} + \hat{j}$.

5-Mark Questions

1. Find the angle and vector product between
$$\mathbf{A} = 2\hat{i} - \hat{j} + \hat{k}$$
and
$$\mathbf{B} = \hat{i} + 2\hat{j} - \hat{k}.$$

2. Find the area of the triangle formed by
$$\mathbf{A} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$$
and
$$\mathbf{B} = \hat{\imath} - \hat{\jmath} + 2\hat{k}.$$
3. Evaluate
$$[\mathbf{A}\,\mathbf{B}\,\mathbf{C}]$$
for
$$\mathbf{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \quad \mathbf{B} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \quad \mathbf{C} = \hat{\imath} - \hat{k}.$$
4. Find $\nabla\phi$ and the directional derivative of
$$\phi = x^2y + yz^2$$
at $(1, 1, 1)$ in the direction
$$2\hat{\imath} - \hat{\jmath} + 2\hat{k}.$$
5. Find $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$ for
$$\mathbf{A} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}.$$
6. Determine whether
$$\mathbf{A} = 2xy\hat{\imath} + x^2\hat{\jmath}$$
is irrotational and find its scalar potential if it exists.
7. Find
$$\nabla^2\phi$$
for
$$\phi = x^2y + y^2z + z^2x.$$

8-Mark Questions

1. For
$$\mathbf{A} = (x^2 + yz)\hat{\imath} + (y^2 + zx)\hat{\jmath} + (z^2 + xy)\hat{k},$$
calculate $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$ and determine whether the field is solenoidal or irrotational.
2. Find the directional derivative and its maximum value for
$$\phi = x^2y + yz^2$$
at $(1, 2, 1)$.
3. Given
$$\mathbf{A} = (y + z)\hat{\imath} + (z + x)\hat{\jmath} + (x + y)\hat{k},$$
calculate $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$.
4. Find the area of the triangle formed by
$$\mathbf{A} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$$
and
$$\mathbf{B} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$$
and find a unit vector normal to the triangle.

5. Verify numerically that
$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$
for
$$\mathbf{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$$
and
$$\nabla \times (\nabla\phi) = 0$$
for
$$\phi = x^2 + y^2 + z^2.$$

UNIT-III: VECTOR INTEGRATION AND DIRAC DELTA FUNCTION

1-Mark Questions

1. Write the Jacobian
$$\frac{\partial(x, y)}{\partial(r, \theta)}$$
for
$$x = r\cos\theta, \qquad y = r\sin\theta.$$
2. Write dV in Cartesian coordinates.
3. Write dV in cylindrical coordinates.
4. Write dV in spherical coordinates.
5. State Gauss's divergence theorem.
6. State Stokes' theorem.
7. State Green's theorem.
8. Write the sifting property of
$$\delta(x - a).$$
9. Write
$$\int_{-\infty}^{\infty} \delta(x - a)\, dx.$$
10. State the relation for $\delta(ax)$.

2-Mark Questions

1. Find
$$\frac{\partial(x, y)}{\partial(r, \theta)}$$
for
$$x = r\cos\theta, \qquad y = r\sin\theta.$$
2. Evaluate
$$\int_0^2 x\,\delta(x - 1)\, dx.$$
3. Evaluate
$$\int_{-\infty}^{\infty} (x^2 + 1)\delta(x - 2)\, dx.$$

4. Evaluate
$$\int_0^3 \delta(x-2)\,dx.$$
5. Calculate the flux of
$$\mathbf{A} = x\hat{i} + y\hat{j} + z\hat{k}$$
through a plane surface.
6. Evaluate
$$\int \mathbf{A}\cdot d\mathbf{r}$$
for
$$\mathbf{A} = y\hat{i} + x\hat{j}$$
along a straight line from $(0,0)$ to $(1,1)$.
7. Convert a double integral from Cartesian to polar coordinates.
8. Find
$$\frac{\partial(x,y)}{\partial(u,v)}$$
for
$$x = u+v, \qquad y = u-v.$$
9. Use Gauss's theorem to find the flux of
$$\mathbf{A} = x\hat{i} + y\hat{j} + z\hat{k}$$
through a sphere.
10. Use the sifting property to evaluate
$$\int xe^x\delta(x-1)\,dx.$$

5-Mark Questions

1. Evaluate
$$\iint_R (x^2+y^2)\,dA$$
over a circular region using polar coordinates.
2. Evaluate
$$\int \mathbf{A}\cdot d\mathbf{r}$$
for
$$\mathbf{A} = y\hat{i} + x\hat{j}$$
along the straight-line path from $(0,0)$ to $(1,1)$.
3. Calculate the flux of
$$\mathbf{A} = x\hat{i} + y\hat{j} + z\hat{k}$$
through the sphere
$$x^2+y^2+z^2 = a^2.$$
4. Verify Gauss's divergence theorem for
$$\mathbf{A} = x\hat{i} + y\hat{j} + z\hat{k}$$
over a cube.

5. Verify Stokes' theorem for
$$\mathbf{A} = -y\hat{\imath} + x\hat{\jmath}$$
over the circle
$$x^2 + y^2 = a^2.$$
6. Evaluate
$$\int_{-\infty}^{\infty} (x^2 + 3x + 2)\delta(x - 1)\, dx.$$
7. Using the Gaussian representation, show how $\delta(x)$ is obtained in the limiting process.
8. Evaluate a double integral by changing from Cartesian to polar coordinates.

8-Mark Questions

1. Use Gauss's theorem to calculate the total flux of
$$\mathbf{A} = x^3\hat{\imath} + y^3\hat{\jmath} + z^3\hat{k}$$
through the sphere
$$x^2 + y^2 + z^2 = a^2.$$
2. Verify Stokes' theorem for
$$\mathbf{A} = -y\hat{\imath} + x\hat{\jmath}$$
over the circular boundary
$$x^2 + y^2 = a^2.$$
3. Evaluate a triple integral over a sphere by changing to spherical coordinates and calculating the Jacobian.
4. Evaluate a multiple integral involving
$$\delta(x - a)\delta(y - b)$$
and explain the sifting property.
5. Evaluate
$$\int_{-\infty}^{\infty} f(x)\delta(g(x))\, dx$$
for a specified $g(x)$ with simple roots.
6. Use Green's theorem to evaluate a line integral around a closed plane curve.

UNIT-IV: ORTHOGONAL CURVILINEAR COORDINATES

1-Mark Questions

1. Write
$$x = r \sin\theta \cos\phi$$
in spherical coordinates.
2. Write
$$y = r \sin\theta \sin\phi$$
in spherical coordinates.

3. Write
$$z = r\cos\theta$$
in spherical coordinates.
4. Write the cylindrical transformation
$$x = \rho\cos\phi.$$
5. Write the scale factors for cylindrical coordinates.
6. Write the scale factors for spherical coordinates.
7. Write dV in cylindrical coordinates.
8. Write dV in spherical coordinates.
9. Write the radial component of velocity in spherical coordinates.
10. Name two physical problems where spherical coordinates are useful.

2-Mark Questions

1. Convert the Cartesian point $(1, 1, 1)$ into spherical coordinates.
2. Convert the cylindrical point
$$(2, \pi/2, 3)$$
into Cartesian coordinates.
3. Find the Jacobian of the cylindrical coordinate transformation.
4. Find the Jacobian of the spherical coordinate transformation.
5. Write the line element in cylindrical coordinates.
6. Write the line element in spherical coordinates.
7. Write $\nabla\phi$ in cylindrical coordinates.
8. Write $\nabla \cdot \mathbf{A}$ in cylindrical coordinates.
9. Write $\nabla^2\phi$ in spherical coordinates.
10. Find the distance between two points using a suitable curvilinear coordinate system.

5-Mark Questions

1. Convert the Cartesian point
$$(1, 1, \sqrt{2})$$
into spherical coordinates.
2. Convert the spherical point
$$(2, \pi/4, \pi/3)$$
into Cartesian coordinates.
3. Derive the volume element
$$dV = \rho\, d\rho\, d\phi\, dz$$
in cylindrical coordinates.
4. Derive the volume element
$$dV = r^2 \sin\theta\, dr\, d\theta\, d\phi$$
in spherical coordinates.

5. Find $\nabla\phi$ for
$$\phi = \rho^2 z$$
in cylindrical coordinates.
6. Find $\nabla \cdot \mathbf{A}$ for
$$\mathbf{A} = \rho\,\hat{e}_\rho + z\,\hat{e}_z$$
in cylindrical coordinates.
7. Find $\nabla^2\phi$ for
$$\phi = r^2 \cos\theta$$
in spherical coordinates.
8. Calculate the velocity components of a particle with
$$\rho = t^2, \qquad \phi = 2t, \qquad z = 3t.$$
9. Calculate the velocity components of a particle with
$$r = t, \qquad \theta = t^2, \qquad \phi = 2t.$$
10. Convert and evaluate a volume integral over a sphere using spherical coordinates.

8-Mark Questions

1. Derive the expressions for $\nabla\phi$, $\nabla\cdot\mathbf{A}$ and $\nabla\times\mathbf{A}$ in cylindrical coordinates and apply them to a specified vector field.
2. Derive the expressions for $\nabla\phi$, $\nabla\cdot\mathbf{A}$, $\nabla\times\mathbf{A}$ and $\nabla^2\phi$ in spherical coordinates and apply them to a specified field.
3. A particle has
$$\rho = t^2, \qquad \phi = 2t, \qquad z = 3t.$$
Find its velocity and acceleration in cylindrical coordinates.
4. A particle has
$$r = t, \qquad \theta = t^2, \qquad \phi = 2t.$$
Find its velocity and acceleration in spherical coordinates.
5. Evaluate
$$\iiint_V (x^2 + y^2 + z^2)\, dV$$
over a sphere using spherical coordinates.
6. Evaluate the volume and a scalar-field integral over a cylinder using cylindrical coordinates.